

Kerolos & Mohamed Mohamed

N-Body Gravitational Simulation Using Barnes–Hut and Particle–Mesh Methods

Summary

Section	Maximum + bonus	Your Grade + bonus
1	6+3	4+2
2	25+4	19+0
3	22+5	15+0
4	20+3	8+0
5	4+3	0+0
6	10+3	10+0
Deductions	-10	0
Total	87+21	56+2

Section 1: Introduction (6 points)

Essential Elements (6 points):

- ☒ (1 pt) Clearly states the problem: optimizing N-body simulations beyond brute-force $O(N^2)$
- ☒ (1 pt) Mentions the original 14-body shockwave simulation and its computational limitations
- ☒ (1 pt) Introduces Barnes-Hut algorithm and its core concept (hierarchical tree, center-of-mass approximation)
- ☒ (1 pt) Introduces Particle-Mesh algorithm and its core concept (grid-based, FFT for potential)
- ☐ (1 pt) States the comparison objectives: runtime, accuracy, scalability
- ☐ (1 pt) Previews the methodology (parameter sweeps, multiple N values)

Bonus Elements (up to 4 points):

- ☒ (+1 pt) Discusses computational complexity: $O(N \log N)$ for BH, $O(N + N_g \log N_g)$ for PM

- ☐ (+1 pt) Provides historical/scientific context for these algorithms
 - ☐ (+1 pt) Mentions real-world applications (astrophysical simulations, galaxy formation)
 - ☒ (+1 pt) Clearly articulates research questions or hypotheses
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Section 2: Methods (25 points)

Barnes-Hut Implementation (8 points):

- ☒ (2 pts) Describes quadtree construction and hierarchical space division
- ☒ (2 pts) Explains the opening angle parameter θ and the approximation criterion ($s/d < \theta$)
- ☒ (2 pts) Describes force calculation via tree traversal
- ☐ (1 pt) Mentions tree update frequency (every 5-10 timesteps as specified)
- ☐ (1 pt) Lists θ values tested: [0.3, 0.5, 1.0, 1.5]

Particle-Mesh Implementation (8 points):

- ☒ (2 pts) Describes mesh discretization and grid setup
- ☒ (2 pts) Explains CIC (Cloud-In-Cell) mass interpolation
- ☒ (2 pts) Describes FFT-based Poisson solver: $\nabla^2\Phi = 4\pi G\rho$
- ☒ (1 pt) Explains force computation via gradient (finite differencing) and interpolation back to particles
- ☐ (1 pt) Lists grid resolutions tested: $N_g \in \{32^3, 64^3, 128^3\}$

General Methodology (9 points):

- ☒ (2 pts) States physical constants used ($G = 4\pi^2$, star mass = 1.0, planet mass range)
- ☒ (1 pt) Specifies the integrator used (and justifies choice from previous project)
- ☒ (2 pts) Describes shockwave collision model with equation
- ☒ (1 pt) Specifies simulation parameters (T , dt , or references to testing protocol)
- ☐ (1 pt) Lists metrics measured: runtime, energy drift, angular momentum, collision count
- ☐ (1 pt) Describes the comparison framework ($N = 14, 100, 500$)
- ☐ (1 pt) Mentions number of trials/runs for statistical validity

Bonus Elements (up to 3 points):

- ☐ (+1 pt) Includes pseudocode or algorithm flowcharts
- ☐ (+1 pt) Discusses boundary condition handling
- ☐ (+1 pt) Explains softening parameter or collision detection implementation

Section 3: Results (22 points)

Required Figures (12 points):

- ☒ (3 pts) Energy vs. Time plots (for both algorithms, clear comparison)
- ☒ (3 pts) Angular Momentum vs. Time plots
- ☒ (3 pts) Trajectory/orbit visualizations
- ☐ (3 pts) Runtime comparison (bar or line chart) across N values

Data Presentation (6 points):

- ☐ (2 pts) Numerical results for all 5 cases in the specification table
- ☒ (2 pts) Clear labeling of axes, legends, and units on all figures
- ☐ (2 pts) Quantitative comparison tables (runtime, energy drift, collision counts)

Completeness (4 points):

- ☒ (1 pt) Results for $N = 14$ (both algorithms)
- ☒ (1 pt) Results for $N = 100$ (both algorithms)
- ☒ (2 pt) Parameter sensitivity analysis (how θ or N_g affects results)

Bonus Elements (up to 5 points):

- ☐ (+1 pt) Animated visualizations or time-lapse trajectories
- ☐ (+1 pt) Log-scale plots showing scaling behavior
- ☐ (+1 pt) Direct overlay comparisons with brute-force baseline
- ☐ (+1 pt) Memory usage analysis
- ☐ (+1 pt) Additional N values beyond requirements (e.g., $N = 1000$)

Section 4: Discussion (20 points)

Algorithm Comparison (10 points):

- ☐ (2 pts) Analyzes runtime scaling differences between BH and PM
- ☒ (2 pts) Discusses accuracy trade-offs (energy conservation, trajectory fidelity)
- ☒ (2 pts) Evaluates the effect of θ on BH performance and accuracy
- ☐ (2 pts) Evaluates the effect of grid resolution on PM performance and accuracy
- ☒ (2 pts) Identifies which algorithm is preferred for different scenarios/N ranges

Critical Analysis (6 points):

- ☐ (2 pts) Discusses limitations of each method

- ☐ (2 pts) Addresses unexpected results or anomalies
- ☐ (2 pts) Connects results to theoretical expectations (e.g., $O(N \log N)$ scaling)

Shockwave Handling (4 points):

- ☐ (2 pts) Discusses how each algorithm handles collisions/shockwaves
- ☒ (2 pts) Analyzes collision count differences between methods

Bonus Elements (up to 4 points):

- ☐ (+1 pt) Discusses potential hybrid approaches (e.g., P^3M)
 - ☐ (+1 pt) Compares to published benchmarks or literature
 - ☐ (+1 pt) Discusses parallelization potential
 - ☐ (+1 pt) Provides recommendations for specific use cases
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Section 5: Conclusion (4 points)

Essential Elements (4 points):

- ☐ (1 pt) Summarizes key findings
- ☐ (1 pt) States which algorithm performed best overall (with justification)
- ☐ (1 pt) Addresses the original research objectives
- ☐ (1 pt) Suggests future work or improvements

Bonus Elements (up to 2 points):

- ☐ (+1 pt) Reflects on lessons learned
 - ☐ (+1 pt) Proposes specific extensions (e.g., 3D octree, adaptive mesh refinement)
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Section 6: Appendix & Code Quality (10 points)

- ☒ (3 pts) Code is included and readable
- ☒ (2 pts) Code is well-documented with comments
- ☒ (2 pts) Code organization is logical (separate functions for BH, PM, integrator, etc.)
- ☒ (2 pts) Reproducibility: parameters and seeds are documented
- ☒ (1 pt) Uses Numba or other optimization as specified

Bonus Elements (up to 3 points):

- ☐ (+1 pt) Includes unit tests or validation checks
- ☐ (+1 pt) Provides clear instructions for running the code

- ☐ (+1 pt) Modular, reusable code structure
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Overall Presentation & Writing (Deductions)

- ☐ (-1 to -3 pts) Poor grammar, spelling, or unclear writing
- ☐ (-1 to -2 pts) Inconsistent formatting
- ☐ (-1 to -2 pts) Missing figure captions or references
- ☐ (-1 to -3 pts) Plagiarism or uncited sources