Faissel & Nada

Comparative Analysis of the 2D Ising Model Utilizing Local Update and Cluster Algorithms

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Summary

Section	Maximum + bonus	Your Grade + bonus
1	6+4	5+1
2	25+4	20+2
3	28+4	25+3
4	20+4	16+2
5	4+2	3+0
6	9+2	9+1
Deductions	-11	-7
Total	92+20	71+9

Section 1: Introduction (10 points)

Essential Elements (6 points):

- [x] (1 pt) Introduces the 2D square-lattice Ising model and its significance
- [x] (1 pt) Explains the phase transition at critical temperature $T_c \approx 2.269\ J/\kappa B$
- [x] (1 pt) Describes the problem with single-spin flip methods at low T (getting stuck, ergodicity issues)
- [ ] (1 pt) Introduces the Monte Carlo approach and concept of equilibrium sampling
- [x] (1 pt) Mentions the algorithms to be compared
- [x] (1 pt) States the objective: comparing efficiency and accuracy of different update schemes

Bonus Elements (up to 4 points):

- [x] (+1 pt) Discusses critical slowing down near Tc
- [ ] (+1 pt) Mentions the exact analytical solution (Onsager) as reference
  You mentioned it but didn't actually elaborate on it as a reference.
- [ ] (+1 pt) Explains how $\langle m \rangle = 0$ analytically but simulations show $\pm|m|$

- [ ] (+1 pt) Provides historical context or applications of the Ising model

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Section 2: Methods (25 points)

Metropolis Algorithm (5 points):

- [x] (2 pts) Describes single-spin flip proposal
- [x] (2 pts) Explains acceptance criterion: min(1, e^{-βΔE})
- [x] (1 pt) Defines a "sweep" as N attempted flips

Heat Bath Algorithm (5 points):

- [x] (2 pts) Describes the heat bath update rule
- [x] (2 pts) Explains probability calculation: P(si = +1) = 1/(1 + e^{-2βhi})
- [x] (1 pt) Notes relationship/difference with Metropolis

Wolff Cluster Algorithm (6 points):

- [x] (2 pts) Describes cluster building from random seed
- [x] (2 pts) States the bond probability: padd = 1 - e^{-2βJ}
- [ ] (1 pt) Explains the cluster flip operation
- [ ] (1 pt) Discusses when Wolff may not be optimal (very low T, whole-lattice clusters)

General Methodology (9 points):

- [x] (1 pt) Specifies boundary conditions (periodic)
- [x] (1 pt) Lists system sizes: L ∈ {16, 32, 64, 128} or similar
- [x] (1 pt) Describes temperature grid (coarse far from Tc, fine near Tc)
- [ ] (2 pts) Explains thermalization/burn-in procedure
- [x] (2 pts) Defines observables: ⟨|m|⟩, ⟨e⟩, χ, C, Binder cumulant UL
- [ ] (1 pt) Explains integrated autocorrelation time τ estimation
  You explained that you used an approximate to calculate it. Sure, but it left me with lingering questions like "how did you determine t?"
- [x] (1 pt) Mentions use of identical random seeds for fair comparison

Bonus Elements (up to 4 points):

- [x] (+1 pt) Includes pseudocode for each algorithm
- [ ] (+1 pt) Discusses error estimation with autocorrelation correction
- [ ] (+1 pt) Explains uncertainty
- [x] (+1 pt) Describes Swendsen-Wang or parallel tempering

Section 3: Results (30 points)

Required Figures (18 points):

- [x] (3 pts) CPU time per independent sample vs. Temperature (all algorithms on one plot)
- [x] (3 pts) Integrated autocorrelation time τ vs. Temperature for each algorithm
- [x] (3 pts) ⟨|m|⟩ vs. T (showing phase transition)
- [x] (3 pts) ⟨e⟩, χ, C vs. T plots
- [x] (3 pts) Binder cumulant crossings for multiple L values
- [x] (3 pts) At least one heatmap (CPU time or τ as function of L and T)

Data Presentation (6 points):

- [x] (2 pts) Results for multiple system sizes as specified
- [x] (2 pts) Clear labeling of axes, legends, units
  You didn't mention any units at all in any figures (but I'll let it pass)
- [ ] (2 pts) Error bars accounting for autocorrelation

Algorithm-Specific Results (4 points):

- [x] (2 pts) Cluster size distribution for Wolff algorithm
- [x] (1 pt) Shows divergent behavior of τ or χ near Tc
- [ ] (1 pt) Results from 3-5 independent trials for heatmap cells
  I'm 99% certain you did DO this, yet it isn't mentioned anywhere ever in the paper. The figures are also too small for me to see anything reasonable.

Bonus Elements (up to 4 points):

- [x] (+1 pt) Finite-size scaling analysis
  meh?
- [x] (+1 pt) Tc estimation from Binder cumulant crossings
- [ ] (+1 pt) Comparison with exact/analytical values
  The results section is.... missing the results. They aren't mentioned in the discussion either. 100% of the statements are qualitative, which is nice, but it weakens your paper. I don't want you determining for me if something is "bad" or "good", I want to look at the numbers myself, be like "wow that really is bad", and then hear you explain it. Yes, you have the graphs, but that shouldn't replace directly putting the numbers into your text as *comparison points*.
- [x] (+1 pt) Swendsen-Wang results (if implemented)

Section 4: Discussion (20 points)

Algorithm Comparison (10 points):

- [x] (2 pts) Compares efficiency (CPU time per independent sample) across algorithms
- [x] (2 pts) Analyzes autocorrelation time differences, especially near Tc
- [x] (2 pts) Discusses which algorithm is best in different temperature regimes
- [x] (2 pts) Explains why Wolff reduces critical slowing down
  This is kind of done in the methodology section, which is wrong, but I'll let it pass. Sadly what's stopping your paper from becoming really good is this exact thing. You mention a lot of stuff in places where its early.
- [x] (2 pts) Addresses trade-offs (per-step cost vs. decorrelation)

Physical Interpretation (6 points):

- [x] (2 pts) Interprets the phase transition in simulation results
- [x] (2 pts) Discusses finite-size effects on observables
- [x] (2 pts) Explains behavior of susceptibility and heat capacity peaks

Critical Analysis (4 points):

- [ ] (2 pts) Addresses limitations or unexpected results
  P.S. I mean limitations *you* faced. Unexpected results you couldn't explain. These are the trademarks of a realllly good paper
- [ ] (2 pts) Discusses sources of error and their mitigation

Bonus Elements (up to 4 points):

- [x] (+1 pt) Compares to literature values for Tc or critical exponents
- [ ] (+1 pt) Discusses potential improvements or optimizations
- [ ] (+1 pt) Analyzes parallel tempering results (if implemented)
- [x] (+1 pt) Provides practical recommendations for different simulation scenarios

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Section 5: Conclusion (5 points)

Essential Elements (4 points):

- [x] (1 pt) Summarizes key findings on algorithm performance
- [x] (1 pt) States which algorithm is preferred and under what conditions
- [x] (1 pt) Confirms observation of phase transition at expected Tc
- [ ] (1 pt) Suggests future work or extensions

Bonus Elements (up to 2 points):

- [ ] (+1 pt) Reflects on broader implications for Monte Carlo simulations
- [ ] (+1 pt) Proposes specific extensions (3D Ising?)

Section 6: Appendix & Code Quality (9 points)

- [x] (3 pts) Code is included and readable
- [x] (2 pts) Code is well-documented with comments
- [x] (2 pts) Code organization is logical (separate functions for each algorithm)
- [x] (2 pts) Reproducibility: random seeds, parameters documented

Bonus Elements (up to 2 points):

- [ ] (+1 pt) Includes validation tests (detailed balance check)
- [x] (+1 pt) Efficient implementation (vectorization, Numba, etc.)

Overall Presentation & Writing (Deductions)

- [x] (-1 to -3 pts) Poor grammar, spelling, or unclear writing
  No nomenclature section. Some variables and scientific terms are introduced without any explanation. (-1)
- [ ] (-1 to -2 pts) Inconsistent formatting
- [x] (-1 to -3 pts) Missing figure captions or references
  0 captions + none of them are mentioned anywhere in the text. (-3)
- [x] (-1 to -3 pts) Plagiarism or uncited sources
  Only 1 reference? no way. (-3)